

comparison with the action of the potential wall. To show this let n approach zero in the equation (3), giving:

$$\frac{d^2 R}{d\rho^2} + \frac{2}{\rho} \frac{dR}{d\rho} - \left\{ \frac{1}{4} + \frac{l(l+1)}{\rho^2} \right\} R = 0 \quad (54)$$

or modified by writing $\rho = 2rn^{-1} = 2ir\sqrt{2E}$ (2)

$$\frac{d^2 R}{dr^2} + \frac{2}{r} \frac{dR}{dr} + \left\{ 2E - \frac{l(l+1)}{r^2} \right\} R = 0. \quad (55)$$

This is indeed the equation of the particle in a spherical box. The solutions of (55) are the Bessel-functions

$$J_{l+\frac{1}{2}}(r\sqrt{2E}), \quad (56)$$

that can also readily be found as an asymptotic case of (4) for $E \rightarrow \infty$ as a series expansion shows.

For $l=0$ the Bessel function (56) has nodes at $r_0\sqrt{2E} = q\pi$ where q is an integer. So for the energy curve of the 1s level the cubic hyperbole

$$E = \pi^2/2r_0^2 \quad (57)$$

is found to be an asymptote. For the 2s level it is:

$$E = 2\pi^2/r_0^2, \quad (58)$$

whereas for $l=1$ the first node of (56) lies at $r_0\sqrt{2E} = 4.4934$ so as to give

$$E = 10.0953/r_0^2 \quad (59)$$

for the asymptote of the $2p$ level. All three asymptotes are indicated in figure 3 as dotted lines. The $r_0 = 0$ axis is evidently also an asymptote of the energy curves calculated.

f) By plotting E^{-1} as a function of r_0 , the asymptotes (57), (58) and (59) become straight lines through the origin, being there tangent to the corresponding (E^{-1}, r_0) curves. It is easy to find now graphically points of the (E, r_0) curve.

The results are listed in tables II-IV with the indication § 3/ and represented in figure 3.

g) Although it follows from section *c* of this paragraph that the energy values E for $\lim r_0 \rightarrow 0$ are the same as for a spherical box, it is not allowed to conclude that the quantum mechanical average potential energy \bar{V} is zero, as with the box.

In fact, it is only true that $\lim_{r_0 \rightarrow 0} \bar{V}/E = 0$. Taking the value of

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 $\lim_{r_0 \rightarrow 0} \bar{V}$

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